Finite element modeling of blood flow: 
Relevance to atherosclerosis

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Abstract

Computational methods are emerging as powerful tools for quantifying blood flow in arteries for disease research, medical device design and treatment planning. The motivation for quantifying hemodynamic conditions in the human vascular system is presented. A computational method for modeling blood flow, based on the theory of stabilized finite element methods, is detailed and shown to yield excellent solutions as compared to laser Doppler anemometry experimental flow data in a vascular bypass anastomosis. The blood flow field in an idealized model of the abdominal aorta under resting and exercise pulsatile flow conditions is quantified and the changes in shear stress discussed. Computational methods are applied for vascular surgery planning by considering blood flow in alternative treatments for a case of aorto-iliac occlusive disease. Finally, the significant challenges that remain in applying computational methods to disease research, device design, and treatment planning are discussed.
1. Introduction

In order to prevent, diagnose and treat vascular disease, detailed knowledge of blood flow and the response of blood vessels is essential. The arteries respond to local and global stimuli and adapt to changes in blood flow and blood pressure [2, 13, 16, 18, 26, 31, 34, 60, 80]. However, with the progression of vascular disease, the adaptive and healing processes fail and the arteries are often unable to deliver blood in the requisite amounts and withstand the forces imposed by the blood upon the vessel walls. Hemodynamic (blood fluid mechanic) factors are strongly correlated with the localization of atherosclerotic plaques, which can obstruct the blood vessel and impede blood flow, and in artery wall and plaque degenerative processes leading to aneurysmal disease. It has been widely observed and noted that clinically relevant plaque deposits are most common in areas of complex flow in the coronary, carotid, abdominal, and femoral arteries [6, 15, 29, 43, 79, 81, 82]. These complex flow regions often occur due to branching, bifurcations, and curvature of the arteries.

Zarins et al. [79] through a combined autopsy and experimental flow study demonstrated that in the carotid artery, atherosclerotic lesions localize along the outer wall of the carotid sinus region where wall shear stress is low. Conversely, the inner wall of the carotid sinus, an area of rapid, laminar and axial flow, and high wall shear stress, was observed to be devoid of plaque. Ku et al. [28] extended this work to pulsatile flow and noted strong positive correlations between intimal thickening and the inverse of the maximum wall shear stress, the inverse of the mean wall shear stress and oscillations in shear characterized by an oscillatory shear index.

In the aorta, it is observed that atherosclerotic disease develops first in the abdominal aorta, and is much more common in the abdominal aorta than the thoracic aorta. Roberts et al. [55] noted that the abdominal aorta
contained by far the most severe aortic atherosclerosis with the greatest involvement occurring below the celiac artery. Glagov et al. [17] noted that abdominal aortic atherosclerosis was greater than thoracic aortic atherosclerosis in the majority of cases for males and females, normotensives and hypertensives. Cornhill et al. [8], in examining fatty streaks in young subjects, noted greater involvement along the lateral and posterior walls of the abdominal aorta. Friedman et al. [15] noted increased intimal thickness in regions of low wall shear stress along the lateral walls of the abdominal aortic bifurcation. Moore et al. [43] measured intimal thickness in the distal abdominal aorta in subjects with minimal atherosclerotic disease and noted a positive correlation between oscillatory shear and intimal thickness and a negative correlation between mean shear stress and intimal thickening.

In surgical reconstructions, such as end-to-side anastomoses of vascular bypass grafts, blood flow can be highly complex [1, 35]. One of the primary causes of long term graft failure, intimal thickening, is attributed in part to changes in hemodynamic and biomechanical conditions including the mismatch between the compliance of the host artery and graft, flow recirculation and relatively low wall shear stress. Presumably, with adequate knowledge of the hemodynamic consequences of surgical procedures, improved designs or techniques can be developed to minimize flow disturbances, and thus maximize long-term graft patency.

Clearly, to examine the relationship between vascular disease and hemodynamic conditions, detailed quantitative data on flow conditions in the arteries is required. Within the last two decades, experimental investigations into the mechanics of blood flow in the vascular system have made significant contributions to the understanding of vascular disease and the long-term consequences of surgical repair. However, experimental model flow studies have several limitations including the time and expense of conducting these experiments, the difficulty in replicating in vivo conditions, and the limited quantitative flow data which can be extracted. Magnetic resonance (MR) imaging and Doppler ultrasound are experimental techniques which can be used to noninvasively quantify blood flow in vivo. These methods have important applications in the diagnosis of cardiovascular disease, research into
disease mechanisms, and to provide input data and validate assumptions and predictions of theoretical models [4, 36, 37, 38, 42, 44, 50].

In recent years, computational techniques have been used increasingly by researchers seeking to understand vascular hemodynamics. These methods can augment the data provided by in vitro and in vivo methods by enabling a complete characterization of hemodynamic conditions under precisely controlled conditions. Application of these methods to flow in the carotid bifurcation and bypass grafts has provided significant information on vascular hemodynamics. Perktold et al. [51] used a finite element method to simulate the pulsatile flow of a Newtonian fluid in a model of a carotid artery bifurcation using a rigid walled approximation. Detailed results on the velocity, pressure and wall shear stress were presented. Lei et al. [32] describe the hemodynamic conditions in a model of a rabbit aorto-celiac junction and postulate a role for the wall shear stress gradient in atherogenesis. Numerical methods are also well suited to the investigation of phenomena difficult to describe using in vitro techniques including wall compliance, mass transport, particle residence time and geometric variations. In an investigation of the effect of wall compliance on pulsatile flow in the carotid artery bifurcation, Perktold and Rappitsch [53] describe a weakly-coupled fluid-structure interaction finite element method for solving for blood flow and vessel mechanics. Steinman and Ethier [64] investigated the effect of wall distensibility in a two-dimensional end-to-side anastomosis. Rappitsch and Perktold [54] describe the transport of albumin in a model of a stenosis. Kunov et al. [30] proposed a new method for describing particle residence time. Perktold et al. [52] examined the effect of bifurcation angle on hemodynamic conditions in carotid artery bifurcation models. Lei et al. [33] describe the application of computational methods to the design of end-to-side anastomoses. Recently, investigators have even started to apply computational methods to models constructed directly from medical imaging data, most notably magnetic resonance imaging [37, 38].

This chapter describes a computational method for simulating blood flow in arteries with applications in disease research and vascular surgical planning. This chapter is organized in the following manner: Section 2 presents the finite element method employed and discusses some of the key developments and issues in system integration, modeling, mesh
generation, finite element analysis, and scientific visualization. Section 3 describes finite element solutions for pulsatile flow in (i) a model of an end-to-side anastomosis and comparison with experimental data, (ii) a model of an abdominal aorta under resting and exercise conditions, and (iii) models of alternative treatment plans for aorto-iliac occlusive disease based on a preoperative model constructed from imaging data. Section 4 comprises the discussion, and Section 5 the concluding remarks.

2. Methods

Taylor et al. developed a software system, the Stanford Virtual Vascular Laboratory, to integrate the model construction, mesh generation, problem initialization, three-dimensional pulsatile flow solution, scientific visualization, and quantitative data extraction aspects of computational vascular modeling [72]. This system was designed to incorporate functionality to create an object model from a list of models including carotid arteries, bifurcations, abdominal aorta, thoracic aorta, end-to-side vascular grafts, and others. All of the models are parametrically defined and hence all of the parameters can be changed to create a new artery model. Analysis objects can be selected to be a subset of the anatomic model. In addition to flow models, methods to simultaneously create a consistent geometric model of the blood vessel wall were implemented. Boundary conditions are defined on the geometric model and are thus automatically updated when this geometry changes. Full functionality exists to initialize the pulsatile flow boundary conditions given a volume flow waveform. The appropriate region to mesh is automatically selected and default mesh refinement parameters specified based on local vessel sizes. Local changes in the mesh refinement can be specified to further resolve features in a particular region of interest such as in the neighborhood of branches. The complete set of input files for a finite element solver are generated automatically. Finally, the finite element solver itself can be started and, upon completion of the analysis, a scientific visualizer employed. This system was developed primarily for solving blood flow solutions in idealized models.

Computational modeling of blood flow requires solving, in the general case, three-dimensional, transient flow equations in deforming
blood vessels. The appropriate framework for problems of this type is the arbitrary Lagrangian-Eulerian (ALE) description of continuous media in which the fluid and solid domains are allowed to move to follow the distensible vessels and deforming fluid domain [22]. The assumption of zero wall motion was utilized for the computations presented herein. Under this assumption, the ALE description of incompressible flow in a deforming fluid domain reduces to the Eulerian description of a fixed spatial domain.

The strong form of the problem governing incompressible, Newtonian fluid flow in a fixed domain consists of the Navier-Stokes equations and suitable initial and boundary conditions. This problem is stated as follows for a domain \( \Omega \subset \mathbb{R}^{n_{sd}} \) where \( n_{sd} \) is the number of spatial dimensions:

Given \( f: \times (0,T) \to \mathbb{R}^{n_{sd}} \), \( g: \Gamma_g \times (0,T) \to \mathbb{R}^{n_{sd}} \), \( h: \Gamma_h \times (0,T) \to \mathbb{R}^{n_{sd}} \) and \( u_0: \Omega \to \mathbb{R}^{n_{sd}} \), we seek \( u(x,t) \), and \( p(x,t) \) \( \forall x \in \Omega \), \( \forall t \in [0,T] \) such that

\[
\rho \left( \frac{\partial u}{\partial t} + (u \cdot \nabla)u \right) = \nabla \cdot \sigma + f, \quad (x,t) \in \Omega \times (0,T) \\
\nabla \cdot u = 0, \quad (x,t) \in \Omega \times (0,T) \\
u(x,0) = u_0(x), \quad x \in \Omega \\
u = g, \quad (x,t) \in \Gamma_g \times (0,T) \\
t = h, \quad (x,t) \in \Gamma_h \times (0,T)
\]

where \( u \) is the velocity, \( p \) is the pressure, \( \rho \) is the density, \( f \) is the body force per unit volume, \( \sigma = \rho I + 2\mu \text{symm}(\nabla u) \) is the stress tensor for a Newtonian fluid, and \( t = \sigma n \) is the traction vector. It is assumed that the initial velocity field \( u_0 \) is divergence-free.

Stabilized finite element methods were introduced by Hughes and colleagues to correct deficiencies in the standard Galerkin finite element method applied to advection dominated flows [5, 23, 24]. The essential features of stabilized methods in the context of incompressible flows are the simultaneous stabilization of the advection operator and the circumvention of the Babuska-Brezzi inf-sup condition restricting the use of many convenient interpolations, including the linear velocity, linear pressure interpolations used in the present work. The basic idea is to
augment the Galerkin finite element formulation with a least squares form of the residual, including appropriate stabilization parameters. These stabilization parameters are designed so that the method achieves exact solutions in the case of one-dimensional model problems involving, for example, the steady advection-diffusion equation.

The stabilized method employed in the present work is described as follows. We start with the mathematical problem of incompressible flow and define the finite element function spaces

\[
V_h = \left\{ v, t \in C^0(\Omega) \mid v|_K \in [R_1(K)]^{\text{nd}} \forall K \in T_h, \ v = 0 \text{ on } \Gamma_g \times (0,T) \right\},
\]

\[
S_h = \left\{ u, \tau \in C^0(\Omega) \mid u|_K \in [R_1(K)]^{\text{nd}} \forall K \in T_h, u = g \text{ on } \Gamma_g \times (0,T) \right\},
\]

\[
P_h = \left\{ p, t \in C^0(\Omega) \cap L^2(\Omega) \mid p|_K \in R_1(K) \forall K \in T_h \right\},
\]

where \( R_1(K) \) is the first order polynomial space on \( K \). The semi-discrete Galerkin method is: Find \( u \in S_h \) and \( p \in P_h \) such that \( \forall w \in V_h, \; q \in P_h \)

\[
B(w, q; u, p) = 0
\]

\[
B(w, q; u, p) = \left( w, \rho \frac{\partial u}{\partial t} + \rho u \cdot \nabla u \right)_\Omega - (\nabla \cdot w, p)_\Omega + (\nabla w, \mu \nabla u)_\Omega - (\nabla q, \rho u)_\Omega - (w, h)_\Gamma - (w, f)_\Omega + (q, \rho u_n)_\Gamma
\]  

(3)

With eqn (3) as a starting point we define our stabilized method as: Find \( u \in S_h \) and \( p \in P_h \) such that \( \forall w \in V_h, \; q \in P_h \)

\[
B_S(w, q; u, p) = 0
\]

\[
B_S(w, q; u, p) = B(w, q; u, p) + (w, \rho (\bar{u} - u) \cdot \nabla u)_\Omega + \left( \rho u \cdot \nabla w + \rho \nabla q, \tau_1 \left( \rho \frac{\partial u}{\partial t} + \rho u \cdot \nabla u + \nabla p - f \right) \right)_\Omega + \left( \rho \nabla \cdot w, \tau_2 \nabla \cdot u \right)_\Omega + \left( \rho (\bar{u} - u) \cdot \nabla w, \tau_3 (\rho (\bar{u} - u) \cdot \nabla u) \right)_\Omega
\]  

(4)
with

\[ \tau_1 = \frac{C}{\sqrt{c_1 \rho^2 u_i g_{ij} u_j + c_2 v^2 g_{ij} g_{ij} + c_3 \frac{\rho}{\Delta t^2}}} \]  

(5)

\[ \tau_2 = \frac{\mu_i g_{ij} u j}{\rho \text{ tr}(g_{ij})} \]  

(6)

\[ \tau_3 = \frac{C}{\sqrt{c \rho^2 (u_i - \bar{u}_i) g_{ij} (u_j - \bar{u}_j)}} \]  

(7)

and

\[ \bar{u} = u - \tau_1 \rho \left( \frac{\partial u}{\partial t} + u \cdot \nabla u + \frac{\nabla p}{\rho} - \frac{f}{\rho} \right) \]  

(8)

where \( g_{ij} \) is the covariant metric tensor, \( C \) is a constant depending on the element topology, and \( c_1, c_2, \) and \( c_3 \) are constants defined from one-dimensional scalar model problems, \( \nu = \mu/\rho, \) and the summation convention for repeated indices is adopted. A second-order accurate time-stepping algorithm is introduced into the above semi-discrete formulation resulting in a nonlinear algebraic problem in each time step. This nonlinear problem is linearized and the resulting linear systems of equations are iteratively solved.

The system of linear equations for the increments in velocity and pressure can be written as:

\[ \begin{bmatrix} \bar{K} & G + Du & \Delta u \\ G^T + D_p & -C & \Delta p \end{bmatrix} = \begin{bmatrix} -R_u \\ R_p \end{bmatrix} \]  

(9)

where \( K \) is a matrix including contributions from inertial, advective, and diffusive terms in eqn (4), \( G \) is the gradient operator, \( G^T \) is the divergence operator, \( C, D_u \) and \( D_p \) arise from the stabilization terms, and \( R_u \) and \( R_p \) are the residuals arising from the momentum and continuity equations,
respectively. Eqn (9) is a nonsymmetric system not amenable to efficient iterative solution. The following procedure is employed. A symmetric form is obtained by neglecting the nonsymmetric terms in eqn (9), viz.,

\[
\begin{bmatrix}
\tilde{K} & G \\
G^T & -C
\end{bmatrix}
\begin{bmatrix}
\Delta u \\
\Delta p
\end{bmatrix} =
\begin{bmatrix}
-R_u \\
R_p
\end{bmatrix}
\tag{10}
\]

where \( \tilde{K} = (1 + \alpha) \text{diag}(K) \) and \( \alpha \) is a positive regularization parameter. We can use eqn (10) to express \( \Delta u \) in terms of \( \Delta p \) as

\[
\Delta u = -\tilde{K}^{-1} (R_u + G \Delta p)
\tag{11}
\]

and then eliminate \( \Delta u \) to obtain an equation for \( \Delta p \):

\[
\left( C + G^T \tilde{K}^{-1} G \right) \Delta p = -R_p - G^T \tilde{K}^{-1} R_u
\tag{12}
\]

This is a symmetric system of linear equations amounting to a discrete Poisson problem that is solved by the Conjugate Gradient method. We then compute \( \Delta u \) by solving a regularized version of the first equation of (9), namely,

\[
(K + \beta K_r) \Delta u = -R_u - (G + D_u) \Delta p
\tag{13}
\]

where \( \beta \) is a positive regularization parameter and \( K_r \) is a diagonal regularization matrix based on the stabilization parameters. Eqn (13) is solved by the Generalized Minimal Residual (GMRES) method introduced by Saad and Schultz [56]. Finally, we update \( u \) by \( u + \Delta u \) and \( p \) by \( p + \Delta p \) and proceed to the next iteration, i.e. linear solve. This process is continued (usually for 2-4 iterations) until an acceptable level of convergence is obtained for the nonlinear problem. Then we advance to the next time step. This solution algorithm results in a robust method of solving incompressible flow problems. Typically, the velocity equation, (13), converges in less than 5 GMRES iterations and the pressure equation, (12), converges in approximately 20 Conjugate Gradients iterations.
A matrix-free GMRES solver developed by Johan et al. [25] was utilized for the parallel solutions presented, substantially reducing memory requirements [85]. For the flow computations performed on a parallel computer, the computational domain was divided into subdomains using a graph partitioning method described by Karypis and Kumar [27]. The nonlinear evolution equations are typically solved over 3 cardiac cycles with 200-400 time steps per cardiac cycle. Analyses run with a greater number of time steps showed no observable differences in the solution.

After the calculation of the primary variables, \( u \) and \( p \), the solution is post-processed and derived quantities are calculated. The traction vector is computed from the stress tensor, \( \sigma \) and surface normal vector, \( n \), by the relation \( t = \sigma n \) and then the surface traction vector, \( t_s \), defined as the tangential component of the traction vector, is computed from \( t_s = t - (t \cdot n)n \).

We define the mean shear stress, \( \tau_{mean} \), a scalar quantity, as the magnitude of the time-averaged surface traction vector as

\[
\tau_{mean} = \frac{1}{T} \int_0^T t_s \, dt
\]  

and define the absolute shear stress, \( \tau_{abs} \), another scalar quantity, as the time-averaged magnitude of the surface traction vector as

\[
\tau_{pulse} = \frac{1}{T} \int_0^T |t_s| \, dt
\]  

Following He and Ku [19], we define the oscillatory shear index (OSI) as

\[
\text{OSI} = \frac{1}{2} \left(1 - \frac{\tau_{mean}}{\tau_{abs}}\right)
\]  

Note that in the special case of the steady, uniaxial flow of a Newtonian fluid in a circular cylinder, the magnitude of the surface traction reduces to the wall shear stress, \( \tau_w = 4\mu Q/\pi r^3 \) where \( \mu \) is the
viscosity, $Q$ is the flow rate, and $r$ is the lumen radius. However, for complex flow fields, the surface traction is a more general measure of surface forces as it is a vector quantity and accounts for secondary flow phenomena.

3. Results

This section describes the application of the computational methods to problems in blood flow. The first example is pulsatile flow in a model of an end-to-side graft-artery anastomosis. The computed pulsatile flow results are compared with published experimental data. Detailed results on steady and pulsatile flow can be found in Taylor et al. [72]. The second example is pulsatile flow in an idealized model of an abdominal aorta under simulated resting and exercise conditions. Further results for resting conditions can be found in Taylor et al. [73] and for exercise conditions in Taylor et al. [74]. The final example presented, the evaluation of alternate surgical procedures for a case of aorto-iliac occlusive disease, illustrates the application of the methodology to vascular surgical planning. The application of computational methods for vascular surgery planning is discussed further in Taylor et al. [75].

3.1 Pulsatile flow in an end-to-side anastomosis

The construction of an end-to-side vascular bypass graft is frequently employed by surgeons to bypass a diseased or injured segment of an artery. Hemodynamic conditions, including low shear stress, are hypothesized to be one of the factors in the localization of the intimal thickening observed at the toe and floor of the anastomosis. Computational simulations of blood flow in graft-artery anastomoses can provide a means to quantify the hemodynamic conditions for comparison with observed sites of intimal thickening and ultimately design anastomoses that minimize adverse hemodynamic conditions.

Loth [35] describes the construction of an in vitro end-to-side graft model and velocity fields measured with LDA under a variety of steady and pulsatile flow conditions. Using fluid dynamic similarity principles, the data from the in vitro model was scaled to match an in vivo canine
ilio-femoral anastomoses model. The model geometry for the present computational study was constructed so as to correspond to the dimensions of this \textit{in vivo} model and is depicted in Figure 1. A symmetry condition was utilized to reduce computing requirements, so only one half of the model was discretized into 21,150 nodes and 103,800 tetrahedral elements. The maximum element size was specified to be \(1/15\)th of the host artery diameter.

![Figure 1: Geometric solid model of the end-to-side anastomosis model. From Taylor \textit{et al.} [72].](image)

The flow conditions were specified to correspond to the pulsatile flow experiments of Loth [35] scaled to \textit{in vivo} conditions: namely, a graft input flow rate of 1.9 ml/sec, viscosity of 0.035 g/sec/cm, and density of 1.05 g/ml. A Womersley profile was specified at the graft inlet to achieve the desired input flow waveform. The division of flow between the proximal and distal outlet segments was set at 20:80 by prescribing a Womersley outflow velocity profile at the proximal outlet segment (POS) and setting the pressure at the distal outlet segment (DOS) to zero.

The flow patterns under pulsatile flow conditions are far more complex than those observed during steady flow conditions. Vortices form at different locations in the anastomotic region and are convected and dissipated during the cardiac cycle [72].
Figure 2 shows the comparison between the computed velocity profiles and experimental data from Loth [35] for the x components of velocity along a line through the midplane in the plane of the anastomosis near the proximal end of the anastomotic region. Velocity is plotted as a function of phase angle in the cardiac cycle. It is observed that the computational results compare favorably qualitatively and quantitatively with the experimental data.

Figure 2: Computed and experimental velocity components in x direction at a cross-section near the proximal end of the anastomotic region. From Taylor et al. [72].
Figure 3 shows the comparison between the computed velocity profiles and experimental data from Loth [35] for the x components of velocity along a line through the midplane in the plane of the anastomosis near the center of the anastomotic region. The comparison between computational and experimental data at this location is also very good for all times in the cardiac cycle. Taylor et al. [72] provides additional validation of the numerical methods for steady and pulsatile flow.

Figure 3: Computed and experimental velocity component in x direction at a cross-section near the center of the anastomotic region. From Taylor et al. [72].
This example demonstrates that the pulsatile flow in an anatomically accurate model of an end-to-side artery-graft anastomosis can be computed accurately, given known geometry and flow conditions. Thus, the computational methods developed have the potential to be a useful tool for the design of graft configurations which minimize energy dissipation, flow recirculation, and flow stasis.

3.2 Pulsatile flow in the abdominal aorta

The aorta is the largest artery in the human body and performs the critical function of transferring oxygenated blood from the left ventricle of the heart to the arteries. The abdominal aorta is the segment of the aorta from the level of the diaphragm to the aortic bifurcation just below the navel. Clinically, it is observed that atherosclerotic disease develops first in the abdominal aorta, and is much more common in the abdominal aorta (i.e. below the diaphragm) than the segment of the aorta above the diaphragm, the thoracic aorta. It has been hypothesized that this is due in part to "adverse hemodynamic conditions" such as low shear stress and high particle residence time resulting from the complex flow patterns in the abdominal aorta [29, 39-43, 82].

The anatomic dimensions of the idealized abdominal aorta model, shown in Figure 4, were obtained primarily from Moore et al. [39]. In the model utilized for the present investigations, the aorta tapers from a circular cross-section with diameter of 2.54 cm at the diaphragm to a circular cross-section with diameter of 1.72 cm at the inferior mesenteric artery and then tapers uniformly to an elliptical cross-section with a major axis of 1.53 cm. and minor axis of 1.3 cm. at the aortic bifurcation.
Figure 4: Abdominal aorta model with branches identified. The model is asymmetric with respect to the mid-sagittal plane and tapers from the diaphragm to the bifurcation. From Taylor et al. [73].

It should be noted that the model constructed is not symmetric about the mid-sagittal plane, but rather includes the feature that the left renal artery is located inferior to the right renal artery. A finite element mesh with 268,563 tetrahedral elements and 58,151 nodes was generated using an automatic mesh generator [59].

Under resting conditions, approximately 70% of the blood that enters the abdominal aorta is extracted by the celiac, superior mesenteric, and renal arteries. The majority of the remaining 30% flows down the infrarenal segment through the bifurcation into the legs. This situation reverses under lower limb exercise conditions as the blood is diverted from the viscera to the legs. In contrast to steady flow studies, in vitro and computational studies of pulsatile blood flow require the specification of flow waveforms as well as mean flow rates. The flow rates as a function of time used in the present study for the inflow and branch vessels are shown in Figure 5. The flow rate waveforms shown in figure 5 for the renal artery and iliac artery are those for each of the right and left renal and iliac arteries, respectively. The suprarenal and infrarenal flow rate
time functions and the celiac, superior mesenteric, renal and inferior mesenteric mean flow rates were obtained from Moore and Ku [40]. The flow rate waveforms in the celiac, superior mesenteric, renal and inferior mesenteric arteries were computed to conserve mass and yield the mean flow rates given by Moore and Ku [40].

Figure 5: Flow rate time functions for resting and exercise conditions. Note the triphasic nature of the flow waveforms and the different ordinate scales for each plot. From Taylor et al. [73].

Note that under resting and exercise conditions the abdominal aorta inflow and renal artery outflows are always positive, whereas under resting conditions the iliac flow waveform exhibits the triphasic character
of \textit{in vivo} measurements of infrarenal aortic flow. Based on the volume flow waveforms shown in Figure 5, pulsatile flow velocity boundary conditions, derived from Womersley theory, were prescribed for the inflow boundary and all outflow boundaries excluding the left and right iliac outflow boundaries, where zero pressure boundary conditions were prescribed \cite{78}. It should be noted that the Womersley boundary conditions are time-dependent, axisymmetric velocity profiles at the inlet and outlet boundaries.

Under resting conditions, regions of low ($< 1$ dyne/cm$^2$) mean wall shear stress were observed along the posterior wall opposite the celiac and superior mesenteric arteries and below the renal artery branches along the lateral and posterior walls. Under light exercise conditions a single region of low mean wall shear stress was observed along the posterior wall at the level of the renal arteries. No regions of low mean wall shear stress were observed under moderate exercise conditions.

Figure 6 displays the mean surface traction vectors along the posterior wall of the abdominal aorta. It is noted that the direction of the surface traction vectors is primarily from superior to inferior with the exception of the surface traction vectors on the aorta wall in the neighborhood of the branch vessels. It is noted that for the two sites of relatively low mean wall shear stress along the posterior wall under resting conditions, namely, opposite to the celiac and superior mesenteric arteries and distal to the renal arteries, that the direction of the mean surface traction vectors is predominantly circumferential. Further, opposite the celiac and superior mesenteric arteries the mean surface traction vectors are from posterior to anterior whereas the opposite is true distal to the renal arteries where the mean surface traction vectors are from anterior to posterior. It is also of interest that the mean surface traction vectors form concentric rings around the ostia of the renal vessels as is apparent for the left renal artery in Figure 6.
Figure 6: Mean surface traction vectors along the posterior wall of the abdominal aorta. Note the circumferential orientation of the mean surface traction vectors along the posterior wall of the aorta in the neighborhood of the renal arteries. From Taylor et al. [73].

Mean wall shear stress, $\tau_{\text{mean}}$, is plotted as a function of arc length in Figures 7 and 8 along the anterior and posterior walls, respectively. It is noted that, along the anterior wall, gaps in the plots of shear stress correspond to the location of the branch vessels.
Figure 7: Mean shear stress along the anterior wall from the diaphragm to the aortic bifurcation. From Taylor et al. [74].

It is also observed that mean shear stress increases significantly in the neighborhood of the celiac, superior mesenteric and inferior mesenteric vessels. Under resting, light exercise and moderate exercise conditions the mean shear stress along the anterior wall in the distal infrarenal aorta exceeds the shear stress at the level of the diaphragm. It is observed that under resting conditions the mean shear stress approaches zero along the posterior wall opposite the celiac and superior mesenteric vessels and approximately 2 cm distal to the left renal artery. Under light exercise conditions a single region
of low mean wall shear stress appears immediately distal to the left renal artery. Under moderate exercise conditions, no regions of low mean wall shear stress are observed along the posterior wall of the abdominal aorta. It is also observed that under all flow conditions the mean shear stresses increase in the distal infrarenal abdominal aorta to a level exceeding that at the level of the diaphragm.

![Image of the abdominal aorta and diaphragm]

Figure 8: Mean shear stress along the posterior wall from the diaphragm to the aortic bifurcation. From Taylor et al. [74].

The oscillatory shear index (OSI) is plotted as a function of arc length in Figures 9 and 10 along the anterior and posterior walls, respectively. Examination of the OSI along the anterior wall under resting conditions
reveals a sharp increase of the OSI from zero in the suprarenal aorta to approximately 0.3 immediately distal to the left renal artery and then a gradual decline to a value of approximately 0.05 at the level of the aortic bifurcation.

Figure 9: Oscillatory shear index (OSI) along the anterior wall from the diaphragm to the aortic bifurcation. From Taylor et al. [74].

Under light exercise conditions, the OSI is zero along the anterior wall except for immediately distal to the left renal artery where it...
increases from zero to approximately 0.05. Under moderate exercise conditions the OSI is zero along the anterior wall of the abdominal aorta. The OSI along the posterior wall under resting conditions increases from zero in the suprarenal aorta to approximately 0.4 opposite the superior mesenteric artery, decreases to less than 0.1 at the level of the renal arteries, increases to a maximum value of 0.47 distal to the renal arteries and then decreases to a value of approximately 0.1 at the level of the aortic bifurcation.

Figure 10: Oscillatory shear index (OSI) along the posterior wall from the diaphragm to the aortic bifurcation. From Taylor et al. [74].
Under light exercise conditions, the OSI along the posterior wall increases from zero in the suprarenal aorta to approximately 0.45 at the level of the left renal artery and then decreases to below 0.1 for the remainder of the infrarenal abdominal aorta. Under moderate exercise conditions the OSI is zero along the posterior wall of the abdominal aorta except for immediately distal to the renal arteries where it attains a value of approximately 0.05.

3.3 Vascular surgical planning

In addition to the investigation of the role of hemodynamic factors in vascular adaptation and disease and the design of cardiovascular devices, computer simulation technology can also be applied to the planning of medical treatments, e.g. the design of vascular reconstructions to provide improved blood flow and reduce pressure losses [75]. These techniques can be performed based on average, idealized, models as in Lei et al. [33] who applied computational methods to the design of end-to-side anastomoses. Alternatively, these methods can be applied to an individual using patient-specific models. As applied to an individual patient, a physician would use diagnostic data to reconstruct a model of an individual's vascular anatomy and physiology, and then use simulation techniques to predict the response of that patient to alternative treatments. Image segmentation and image-based geometric modeling techniques can be used to construct patient-specific anatomic models from 3D imaging techniques including CT and MRI [68, 47, 48, 76, 77]. Patient specific physiologic data is also needed and involves extracting flow data to define the preoperative conditions. MRI can potentially be used to define preoperative conditions, but the definition of postoperative conditions requires a representation of the preoperative resistance or impedance of the vascular bed. Once the patient-specific model is created, the predictions of the consequences of treatments requires the simulation of blood flow using techniques from computational fluid dynamics in models which reflect the proposed treatments.

Taylor and colleagues have developed a prototype software system for vascular surgery planning using computational methods for modeling blood flow [75]. This system, the Advanced Surgical Planning Interactive Research Environment (ASPIRE) was developed using the Java
programming language and the Virtual Reality Modeling Language. A typical treatment planning session starts by examining a case presentation. Patient history, medical imaging data and vascular lab data are examined through a web browser. The next step in the treatment planning process is the specification of a treatment plan and involves drawing a treatment on a sketchpad as shown in Figure 11. The sketch is referenced to a three-dimensional computer model reconstructed from medical imaging data. In the present case circular and elliptical curves were fitted to magnetic resonance angiography (MRA) imaging data to extract the luminal surface of the blood vessels. Recently, a geometric segmentation technique, the level set method, has been employed to create patient specific models [76, 77]. Once a treatment plan is created, the next step is to predict the changes in blood flow which result from this treatment. The user submits the treatment to the system for evaluation and computational fluid dynamics calculations are used to compute the three-dimensional flow field and store the information in the system database. The user interface is then operated in the treatment evaluation mode to query the solution and examine the predicted flow distribution and pressure data using graphical means or three-dimensional visualizations.

In order to test the concept of using computational methods for vascular surgery planning, we created a mock clinical case and applied our simulation-based medical planning methodology. The base anatomy for this case was generated by extracting the luminal boundaries from a normal subject.

This subject was injected with a contrast agent (Gd-DPTA) and then imaged using a phase contrast MR angiographic sequence from the chest down to the feet using a 1.5T GE Signa system. A total of 384 slices were acquired in three scans of 128 slices, with in-plane resolution of 0.9375 mm and slice thickness of 1.8 mm.
Figure 11: ASPIRE user interface in treatment planning mode. Shown on the far left are a variety of buttons to query the model and implement treatment options. These treatment options are sketched on a 2D projection of a preoperative model. A treatment plan is sketched for angioplasty with a femoral to femoral bypass graft. A graphical panel is shown to the right of the surgical plan. This panel allows for the extraction of quantitative comparisons of pressure, velocity and volume flow at standard or user-defined measurement locations. The pop-up-menu used to enter graft properties is also shown in the center. Shown in the two panels on the far right are three-dimensional models constructed from preoperative MRI data. Preoperative anatomy and physiology data can be examined and visualized using the Virtual Reality Modeling Language (VRML).
The mock clinical case created from this base anatomy is that of a man with his right iliac artery completely occluded and the left iliac artery with a 50% reduction in diameter as well as right and left superficial femoral artery disease. There are many different surgical and non-surgical treatments which could be attempted to correct this patient’s clinical problems, but any successful procedure would have to increase the blood flow to the right leg. The preoperative geometric model and three of the most common procedures, given this particular patient’s clinical presentation and observed disease, are shown in Figure 12.

![Figure 12: Geometric models for alternative treatment plans for a case of lower extremity occlusive disease examined. (a) Anatomic description with left iliac artery stenosis and right iliac artery occlusion shown, (b) aorto-femoral bypass graft with proximal end-to-side anastomosis, (c) aorto-femoral bypass graft with proximal end-to-end anastomosis, (d) balloon angioplasty in left common iliac artery with femoral to femoral bypass graft. Adapted from Taylor et al. [75].](image-url)
These procedures were modeled and compared to see which treatment would produce the best hemodynamic outcome for this specific case. A finite element mesh was generated for each of the geometric models. Figure 13 shows the surface of the finite element mesh generated for the case of the aorto-femoral bypass with a proximal end-to-side anastomosis of the graft to the aorta (Figure 12b).

Resting flow conditions were used to assess the blood flow in the foot needed for wound healing. Exercise flow conditions were used to assess the blood flow in the right leg under walking conditions. The boundary conditions for preoperative and postoperative computations were prescribed as follows. First, preoperative analyses under resting and exercise steady flow conditions were performed with a specified volume flow rate through each boundary based on previous flow simulations [74]. This preoperative analysis was used to compute the average pressure distribution at each outflow boundary. Second, a unique resistance value was computed for each outflow boundary based on a relationship between pressure and volume flow rate of the form $P = QR$, where $P$ is the mean pressure, $Q$ is the volume flow rate and $R$ is the resistance to flow. For each outflow boundary, the same resistance value was used for all of the post-operative analyses for each surgical plan. Using this strategy, the volume flow rate $Q$ and pressure $P$ were calculated (not specified) for each of the boundaries for each of the surgical plans. Figure 14 depicts the results for the flow velocity in a plane through the aortic bifurcation for the case of the aorto-femoral bypass with a proximal end-to-side anastomosis of the graft to the aorta (Figure 12b).
Figure 13: Close-up of finite element mesh for aorto-femoral bypass with end-to-side anastomosis. Curvature-based mesh refinement techniques were used to obtain adequate spatial resolution in small branch vessels. From Taylor et al. [75].
Figure 14: Close-up view of proximal aorto-femoral bypass with end-to-side anastomosis. Velocity vectors and contours of velocity magnitude are shown on a slice plane through the anastomosis illustrating extraction of quantitative flow data for a vascular surgical plan.

This system for Simulation-Based Medical Planning was demonstrated at the 1998 Society for Vascular Surgery (SVS) meeting. For this demonstration, this system was applied to predict the effect of multiple alternative treatment plans on blood flow and pressure under resting and exercise conditions for the clinical case of lower extremity vascular disease described above. Four prominent vascular surgeons (all former presidents of the SVS) were selected based on their clinical experience. At the start of the demonstration, the surgeons were presented with the clinical case and asked to use the system to implement and evaluate one of four possible surgical plans. The display from each of the SGI Octane computers used by the vascular surgeons was projected to a large display screen for viewing by the audience of vascular surgeons and affiliated health care professionals as shown in Figure 15.
4. Discussion

Hemodynamic conditions, including velocity, shear, and pressure, play an important role in the modulation of vascular adaptation and the localization of vascular disease. Computational methods can play an important role in the quantification of vascular blood flow, vessel deformation, mass transport, and vascular remodeling. There are numerous areas where these computational methods can be improved including boundary conditions, vessel wall mechanics and blood rheology.

The determination of appropriate boundary conditions and flow waveforms for investigations of vascular hemodynamics warrants further attention. In addition to the specification of flow waveforms, assumptions are required for the variation in velocity at the boundaries where flow is specified. In most investigations of blood flow, velocity profiles, pressure
or normal traction are specified for inflow and outflow boundaries. However in many investigations the flow distribution is unknown \( \textit{a priori} \) and a more appropriate boundary condition involves a relationship between pressure and flow. A resistance boundary condition is the simplest such relationship, but is a crude representation of the real system. New methods for boundary conditions for blood flow computations are emerging, however [45,71].

The effect of aortic compliance was not considered in the present investigation. However, this is an active area of research. In an investigation of the effect of wall compliance on pulsatile flow in the carotid artery bifurcation, Perktold and Rappitsch [53] describe a weakly-coupled fluid-structure interaction finite element method for solving for blood flow and vessel deformation. They concluded that the wall shear stress magnitude decreases by approximately 25% in the distensible model as compared to the rigid model, yet the overall effect on the velocity field was relatively minor. Steinman and Ethier [64], based on a study of a compliant two-dimensional end-to-side anastomosis, concluded that the effects of wall distensibility on the flow field are less pronounced as compared to changes in anatomic dimensions or physiologic conditions. It should be noted that although neglecting wall distensibility may not have a major effect on the primary flow field, the incorporation of wall mechanics in these studies is important for other reasons including the description of the stress environment within the vessel walls and the interaction between deformability and mass transport phenomena. In summary, the use of rigid models for flow studies in idealized models of the human abdominal aorta should be viewed as a first approximation.

A Newtonian constitutive model for viscosity was employed in the present investigation. It is generally accepted that this is a reasonable first approximation to the behavior of blood flow in large arteries. Perktold \textit{et al.} [52] examined non-Newtonian viscosity models for simulating pulsatile flow in carotid artery bifurcation models. They concluded that the shear stress magnitudes predicted using non-Newtonian viscosity models resulted in differences on the order of 10% as compared with Newtonian models.
5. Conclusions

The characterization of the temporal and spatial variations of the velocity field in computational models of the vascular system enables new insights into mechanisms by which hemodynamic factors relate to the disease localization patterns observed in cadavers and in vivo. Numerical studies of blood flow provide further impetus to examine hemodynamics in vivo using magnetic resonance imaging techniques, and to assess the validity of the assumptions made regarding vascular anatomy and physiologic conditions, blood rheology and vessel mechanics. In addition to the application of computational methods to disease research, these techniques can be applied to patient-specific models for vascular surgery planning and to the design of cardiovascular devices. It is presumed that knowledge of the effect of hemodynamic conditions on vascular adaptation and disease, and simulations of the actual hemodynamic conditions in an individual patient, can be used to improve medical care.

References


